

# Multisupergravity from latticized extra dimension

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## Abstract

The construction of the linearized four-dimensional multisupergravity from five-dimensional linearized supergravity with discretized fifth dimension is presented. The one-loop vacuum energy is evaluated when (anti)periodic boundary conditions are chosen for (bosons) fermions, respectively or vice-versa. It is proposed that the relation between discretized M-theory and strings may be found in the same fashion.

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# 1 Introduction

It is known that the earlier attempts to construct multigravity theories [1] were not quite successful. The problem is that for massive tensor theories the consistency issue is not yet completely understood. Such theories normally contain several pathologies like van Dam-Veltman-Zakharov discontinuity [2] or the appearance of ghosts beyond quadratic order. The truncation of KK theory to finite number of massive spin-2 fields is not consistent too [3].

Recently, the interesting approach to multigravity theories was initiated by brane-world picture. Such approach suggests a new way to formulate multigravity where it is possible to check how the consistency of brane-world gravity is kept at discretized level. In ref.[4], related with the deconstruction of extra dimension, a model where several four-dimensional gravities are connected by the link variables has been proposed. By choosing a proper gauge condition, it has been shown that the mass term of the graviton is generated. Almost in parallel, in [5], starting from the linearized Einstein theory in five dimensions and replacing the fifth direction with a lattice, a four-dimensional model with massive multi-graviton has been proposed. This has been generalized by introducing the non-nearest neighbour couplings on the lattice [6]. In the model[5], the one-loop vacuum energy is negative in general but in the model of [6], the vacuum energy can be positive. This may explain the acceleration of the present universe. The consistency of discretized gravity (up to quadratic order), its continuum limit and some solutions have been studied in recent works [7]. The attractive property of multigravity is related with the fact that it may lead to interesting (still acceptable!) modifications of Newton law at large scales.

In this paper, we (super)generalize the models[6], where theory [5] is also included, starting from the linearized supergravity in five dimensions. Of course, the latticized action should respect most of the symmetries of the five-dimensional action. As usually, there is the hope that consistency issue appearing in bosonic version may be resolved within more fundamental, supersymmetric theory. The resulting multisupergravity which includes multi-gravitons and multigravitinos can be obtained by replacing the fifth direction in the linearized supergravity to the discrete lattice.

## 2 Multisupergravity from extra dimension

Before deconstructing supergravity, we consider the five-dimensional linearized supergravity. In the five dimensions, there is no Majorana representation of the  $\gamma$ -matrices and the Rarita-Schwinger field (gravitino) should be complex. Therefore, reducing the five-dimensional supergravity [8] to the four-dimensional one, the obtained supergravity has at least  $\mathcal{N} = 2$  (local) supersymmetry. Furthermore, as there is no chiral fermion in the five dimensions, there is no chiral multiplet. The minimum representation of the multiplet including the graviton is associated with the complex Rarita-Schwinger field and  $U(1)$  vector (gauge) field. The number of bosonic degrees of freedom is  $5(\text{graviton})+3(\text{vector})=8$ . On the other hand, the number of fermionic degrees of freedom coming from complex Rarita-Schwinger field is  $4 \times 2 = 8$  (2 comes from the complexity).

The action of the five dimensional linearized supergravity is given by considering the

perturbation from the flat background which is the vacuum. The explicit form is

$$S = \int d^5x \{ \mathcal{L}_E + \mathcal{L}_{RS} + \mathcal{L}_U \} , \quad (1)$$

$$\mathcal{L}_E = -\frac{1}{2} \partial_\lambda h_{\mu\nu} \partial^\lambda h^{\mu\nu} + \partial_\lambda h^\lambda{}_\mu \partial_\nu h^{\mu\nu} - \partial_\mu h_{\mu\nu} \partial_\nu h + \frac{1}{2} \partial_\lambda h \partial^\lambda h , \quad (2)$$

$$\mathcal{L}_{RS} = i \bar{\psi}_\mu \gamma^{\mu\nu\rho} \partial_\nu \psi_\rho , \quad (3)$$

$$\mathcal{L}_U = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} . \quad (4)$$

Here  $h_{\mu\nu}$  is the graviton and  $h \equiv h^\mu{}_\mu$ . The Rarita-Schwinger field is denoted by  $\psi_\mu$ . The  $U(1)$  field strength  $F_{\mu\nu}$  is given by the gauge (vector) field  $A_\mu$  as usual:  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ . We may introduce the fluctuation  $\mathcal{E}_\mu{}^a$  of the fünfbein field  $e_\mu{}^a$  from the flat background:  $e_\mu{}^a = \delta_\mu{}^a + \mathcal{E}_\mu{}^a$ . In terms of  $\mathcal{E}_\mu{}^a$ , one may rewrite  $h_{\mu\nu}$  as  $h_{\mu\nu} = \eta_{\mu a} \mathcal{E}_\nu{}^a + \mathcal{E}_\mu{}^a \eta_{a\nu}$  (we may choose the gravitational coupling constant  $\kappa$  to be unity here). Here  $\eta_{\mu a} = \delta_\mu{}^b \eta_{ba}$  and  $\eta_{a\mu} = \eta_{ab} \delta_\mu{}^b$ . The metric tensor  $\eta_{ab}$  ( $a, b = 0, 1, 2, 3, 4$ ) in the flat local Lorentz space is given by

$$\eta_{ab} = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} . \quad (5)$$

As the action (2) is written in terms of  $h_{\mu\nu}$ , if one expresses  $h_{\mu\nu}$  with the help of  $\mathcal{E}_\mu{}^a$ , the action is invariant under the local Lorentz transformation

$$\delta \mathcal{E}_\mu{}^a = \omega^{ab} \eta_{b\mu} , \quad (6)$$

where  $\omega^{ab}$  is local parameter with  $\omega^{ab} = -\omega^{ba}$ . Under the transformation,  $h_{\mu\nu}$  is invariant. By the transformation (6), the gauge condition may be chosen

$$\frac{1}{2} h_{\mu\nu} = \eta_{\mu a} \mathcal{E}_\nu{}^a = \mathcal{E}_\mu{}^a \eta_{a\nu} . \quad (7)$$

In the linearized gravity, by using the gauge condition (7), we may forget  $\mathcal{E}_\mu{}^a$ .

The action (1) is invariant under the linearized general coordinate transformation:

$$\delta h_{\mu\nu} = \partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu , \quad \delta \psi_\mu = \delta A_\mu = 0 , \quad (8)$$

local supersymmetry transformation:

$$\delta h_{\mu\nu} = \delta A_\mu = 0 , \quad \delta \psi_\mu = \partial_\mu \eta , \quad (9)$$

and  $U(1)$  gauge transformation

$$\delta h_{\mu\nu} = \delta \psi_\mu = 0 , \quad A_\mu = \partial_\mu \sigma . \quad (10)$$

Here  $\epsilon_\mu$ ,  $\eta$ , and  $\sigma$  are the local parameters for the transformations. Even for the local supersymmetry transformation, the transformations do not mix the different kinds of fields. This is because the local transformations correspond to the inhomogeneous part of the full

transformations. We should note, however, that there is a *global* supersymmetry, which mixes the fields:

$$\begin{aligned}\delta h_{\mu\nu} &= \frac{1}{2} (\bar{\zeta} \gamma_\mu \psi_\nu + \bar{\zeta} \gamma_\nu \psi_\mu - \bar{\psi}_\mu \gamma_\nu \zeta - \bar{\psi}_\nu \gamma_\mu \zeta) , \\ \delta \psi_\mu &= -\frac{i}{4} \gamma^{\rho\sigma} \omega_{\mu\rho\sigma} \zeta + F_{\rho\sigma} (\gamma^{\rho\sigma} \gamma_\mu - 5 \gamma^\rho \delta_\mu^\sigma + 5 \gamma^\sigma \delta_\mu^\rho) \zeta , \\ \delta A_\mu &= 80 (\bar{\zeta} \psi_\mu + \bar{\psi}_\mu \zeta) .\end{aligned}\tag{11}$$

Here  $\zeta$  is a constant spinor which is the parameter of the transformation and the spin connection  $\omega_{\mu\nu\rho}$  is defined by

$$\omega_{\mu\nu\rho} \equiv \frac{1}{2} \{ -e_{\rho a} (\partial_\nu e_\mu^a - \partial_\mu e_\nu^a) - e_{\mu a} (\partial_\rho e_\nu^a - \partial_\nu e_\rho^a) + e_{\nu a} (\partial_\mu e_\rho^a - \partial_\rho e_\mu^a) \} ,\tag{12}$$

and linearized in (11):

$$\omega_{\mu\nu\rho} \sim \frac{1}{2} \{ \partial_\mu h_{\rho\nu} - \partial_\rho h_{\mu\nu} - \eta_{\rho a} \partial_\nu \mathcal{E}_\mu^a + \eta_{\mu a} \partial_\nu \mathcal{E}_\rho^a \} .\tag{13}$$

Furthermore, with the gauge condition (7) one gets

$$\omega_{\mu\nu\rho} \sim \frac{1}{2} \{ \partial_\mu h_{\rho\nu} - \partial_\rho h_{\mu\nu} \} .\tag{14}$$

We now consider the deconstruction by replacing fifth spacelike dimension by discrete  $N$  points, which may be regarded as the one-dimensional lattice. There were many works on realization the supersymmetry on the lattice [9, 10] (for recent progress, see [11]). The interesting idea to put the supersymmetry on the finite lattice has been developed in [10]. The problem comes from the difficulty to realize the Leibniz rule on the lattice. Under the supersymmetry transformation, the variation of the Lagrangian density becomes a total derivative by summing up the variation of the field by the Leibniz rule, then such action is invariant. However, the Leibniz rule does not hold for the difference operator on the lattice in general. Let us consider one-dimensional lattice and denote the difference operator by  $\Delta$ . In general,

$$\sum_n \{ (\Delta \phi_n^{(1)}) \phi_n^{(2)} \phi_n^{(3)} + \phi_n^{(1)} (\Delta \phi_n^{(2)}) \phi_n^{(3)} + \phi_n^{(1)} \phi_n^{(2)} (\Delta \phi_n^{(3)}) \} \neq 0 .\tag{15}$$

Here the point (site) on the lattice is denoted by  $n$  and  $\phi^{(1,2,3)}$ 's are variables (fields) defined on the sites. We should note, however, that it is not difficult to realize the supersymmetry on the lattice for the free theory, since we only require the anti-Hermiticity for  $\Delta$ :

$$\sum_n \{ (\Delta \phi_n^{(1)}) \phi_n^{(2)} + \phi_n^{(1)} (\Delta \phi_n^{(2)}) \} = 0 .\tag{16}$$

A set satisfying (16) has been given in [10]. We consider  $N$  variables,  $\phi_n$ , which may be identified with the fields on a lattice with  $N$  sites. The difference operator  $\Delta$  is defined by

$$\Delta \phi_n \equiv \sum_{k=0}^{N-1} a_k \phi_{n+k} .\tag{17}$$

Here it is assumed  $\phi_{n+N} = \phi_n$ , which may be regarded as a periodic boundary condition. Since

$$\sum_{n=0}^{N-1} \phi^{(1)} \Delta \phi_n^{(2)} = \sum_{n,k=0}^{N-1} \phi_n^{(1)} a_k \phi_{n+k}^{(2)} = \sum_{n,k=0}^{N-1} a_{-k} \phi_{n+k}^{(1)} \phi_n^{(2)} , \quad (18)$$

if

$$a_{-k} (= a_{N-k}) = -a_k , \quad (19)$$

Eq.(16) can be satisfied. Note that there is no nontrivial solution in (19) when  $N = 2$ . In order that  $\Delta$  becomes a usual differentiation in a proper continuum limit, the following condition is usually imposed:

$$\sum_{k=0}^{N-1} a_k = 0 , \quad (20)$$

which is satisfied by (19). The eigenvectors for  $\Delta$  are given by

$$\phi_n^M = \frac{1}{\sqrt{N}} e^{i \frac{2\pi n M}{N}} , \quad M = 0, 1, \dots, N-1 \quad (21)$$

and their corresponding eigenvalues, by

$$\Delta \phi_n^M = i m^M \phi_n^M , \quad i m^M = \sum_{n=0}^{N-1} a_n e^{i \frac{2\pi n M}{N}} . \quad (22)$$

Here  $m^0 = 0$ . Assuming (19), one obtains

$$m^M = -m^{N-M} . \quad (23)$$

Note that  $\phi_n^M$  satisfies the following properties, which may be identified with the conditions of normalization and completeness, respectively:

$$\sum_{n=0}^{N-1} \phi_n^{M*} \phi_n^{M'} = \delta^{MM'} , \quad \sum_{M=0}^{N-1} \phi_n^{M*} \phi_{n'}^M = \delta_{nn'} . \quad (24)$$

$a_n$  can be solved with respect to  $m^M$  by

$$a_n = \frac{i}{\sqrt{N}} \sum_{M=0}^{N-1} m^M \phi_n^{M*} = \frac{i}{N} \sum_{M=0}^{N-1} m^M e^{-i \frac{2\pi n M}{N}} . \quad (25)$$

Then by choosing  $a_n$  properly, one may obtain arbitrary spectrum of  $m^M$  with  $m^0 = 0$ . We should note if  $a_n$  is real

$$m^{N-M} = - (m^M)^* . \quad (26)$$

By combining (26) with (23),  $m^M$  should be real. For instance, if  $N = 3$ ,  $m^2 = -m^1$  and

$$a_0 = 0 , \quad a_1 = -a_2 = \frac{m_1}{\sqrt{3}} . \quad (27)$$

For  $N = 4$ , we have  $m^3 = -m^1$  and find  $m^2 = 0$ . Then

$$a_0 = a_2 = 0 , \quad a_1 = -a_3 = \frac{m_1}{2} . \quad (28)$$

The next step is to deconstruct fifth dimension. In the actions (2-4), first replace  $x^5$ -dependence with  $n$ -dependence ( $n = 0, 1, \dots, N-1$ ), and after that replace the derivative with respect to  $x^5$  by the difference operator  $\Delta$  (17). It is assumed (19). First for the Lagrangian density (2), one gets

$$\begin{aligned} \mathcal{L}_E = & \sum_{n=0}^{N-1} \left[ -\frac{1}{2} \partial_\lambda h_{n\mu\nu} \partial^\lambda h_n^{\mu\nu} + \partial_\lambda h_n^\lambda{}_\mu \partial_\nu h_n^{\mu\nu} - \partial_\mu h_{n\mu\nu} \partial_\nu h_n + \frac{1}{2} \partial_\lambda h_n \partial^\lambda h_n \right. \\ & + \frac{1}{2} (\Delta h_{n\mu\nu} \Delta h_n^{\mu\nu} - (\Delta h_n)^2) - 2(-\Delta B_n^\mu + \partial^\mu \varphi_n) (\partial^\nu h_{n\mu\nu} - \partial_\mu h_n) \\ & \left. + \frac{1}{2} (\partial_\mu B_{n\nu} - \partial_\nu B_{n\mu}) (\partial^\mu B_n^\nu - \partial^\nu B_n^\mu) \right]. \end{aligned} \quad (29)$$

Here and in the following, the four-dimensional index is specified by the Greek characters,  $\mu, \nu = 0, 1, 2, 3$ . In (29),

$$B_{n\mu} = h_{n\mu 5}, \quad \varphi_n = h_{n55}. \quad (30)$$

The action  $S_E = \int d^4x \mathcal{L}_E$  from the Lagrangian density (29) is invariant under transformations with the local parameters  $\xi_n^\nu$  and  $\zeta_n$ :

$$\begin{aligned} h_{n\mu\nu} & \rightarrow h_{n\mu\nu} + \partial_\mu \xi_{n\nu} + \partial_\nu \xi_{n\mu}, \\ B_{n\mu} & \rightarrow B_{n\mu} + \Delta \xi_{n\mu} - \partial_\mu \zeta_n, \\ \varphi_n & \rightarrow \varphi_n - \Delta \zeta_n, \end{aligned} \quad (31)$$

which comes from the general coordinate transformation in (8). For the Lagrangian of the Rarita-Schwinger field (3), we have

$$\mathcal{L}_{RS} = \sum_{n=0}^{N-1} \left\{ i\bar{\psi}_{n\mu} \gamma^{\mu\nu\rho} \partial_\nu \psi_{n\rho} + i\bar{\psi}_{n5} \gamma^{5\nu\rho} \partial_\nu \psi_{n\rho} \right. \quad (32)$$

$$\left. + i\bar{\psi}_{n\mu} \gamma^{\mu 5\rho} \Delta \psi_{n\rho} + i\bar{\psi}_{n\mu} \gamma^{\mu\nu 5} \partial_\nu \psi_{n5} \right\}. \quad (33)$$

The action  $S_{RS} = \int d^4x \mathcal{L}_{RS}$  is invariant under the transformation with  $N$ -local fermionic parameters  $\eta_n$  ( $n = 0, 1, \dots, N-1$ ):

$$\delta \psi_{n\mu} = \partial_\mu \eta_n, \quad \delta \psi_{n5} = \Delta \eta_n, \quad (34)$$

which correspond to the local supersymmetry transformation in (9). With the redefinition

$$\psi'_{n\mu} \equiv \psi_{n\mu} + \frac{1}{2} \gamma_\mu \gamma_5 \psi_{n5}, \quad \psi'_n \equiv \sqrt{\frac{3}{2}} \psi_{n5}, \quad (35)$$

the Lagrangian density (32) can be rewritten as

$$\mathcal{L}_{RS} = \sum_{n=0}^{N-1} \left\{ i\bar{\psi}'_{n\mu} \gamma^{\mu\nu\rho} \partial_\nu \psi'_{n\rho} + i\bar{\psi}'_n \gamma^\nu \partial_\nu \psi'_n - i\bar{\psi}'_{n\mu} \gamma^5 \gamma^{\mu\rho} \Delta \psi'_{n\rho} \right. \quad (36)$$

$$\left. - i\sqrt{\frac{3}{2}} \bar{\psi}'_n \gamma^\rho \Delta \psi'_{n\rho} - i\sqrt{\frac{3}{2}} \bar{\psi}'_{n\mu} \gamma^\mu \Delta \psi'_n - 2i\bar{\psi}'_n \Delta \psi'_n \right\}. \quad (37)$$

In terms of the redefined fields, the transformation (34) is rewritten as

$$\delta\psi'_{n\mu} = \partial_\mu\eta_n + \frac{1}{2}\gamma_\mu\gamma_5\Delta\eta_n, \quad \delta\psi'_n = \sqrt{\frac{3}{2}}\Delta\eta_n. \quad (38)$$

By using the transformation (38), the following gauge condition can be chosen

$$\gamma^\mu\psi'_{n\mu} = 0. \quad (39)$$

Then the action reduces to the sum of the action of the (4-dimensional) Rarita-Schwinger field and that of the Dirac spinor fields:

$$\mathcal{L}_{RS} = \sum_{n=0}^{N-1} \{ i\bar{\psi}'_{n\mu}\gamma^{\mu\nu\rho}\partial_\nu\psi'_{n\rho} + i\bar{\psi}'_n\gamma^\nu\partial_\nu\psi'_n + i\bar{\psi}'_{n\mu}\gamma^5\gamma^{\mu\rho}\Delta\psi'_{n\rho} - 2i\bar{\psi}'_n\Delta\psi'_n \}. \quad (40)$$

Finally from the Lagrangian density (4) of the vector field, we obtain

$$\mathcal{L}_U = -\frac{1}{4}F_n^{\mu\nu}F_{n\mu\nu} - \frac{1}{2}(\partial^\mu\rho_n - \Delta A_n^\mu)(\partial_\mu\rho_n - \Delta A_{n\mu}), \quad (41)$$

This is invariant under the transformation, which corresponds to  $U(1)$  gauge transformation (10):

$$\delta A_{n\mu} = \partial_\mu\sigma_n, \quad \delta\rho_n = \Delta\sigma_n. \quad (42)$$

In (41),

$$\rho_n \equiv A_{n5}. \quad (43)$$

In terms of the redefined fields in (30), (35), and (43), the supersymmetry transformation (11) can be rewritten as

$$\begin{aligned} \delta h_{n\mu\nu} &= \frac{1}{2}(\bar{\zeta}\gamma_\mu\psi'_{n\nu} + \bar{\zeta}\gamma_\nu\psi'_{n\mu} - \bar{\psi}'_{n\mu}\gamma_\nu\zeta - \bar{\psi}'_{n\nu}\gamma_\mu\zeta) \\ &\quad + \frac{1}{\sqrt{6}}\eta_{\mu\nu}(-\bar{\zeta}\gamma_5\psi'_n + \bar{\psi}'_n\gamma_5\zeta), \\ \delta B_{n\mu} &= \frac{1}{\sqrt{6}}(\bar{\zeta}\gamma_\mu\psi'_n - \bar{\psi}'_n\gamma_\mu\zeta) + \frac{1}{2}(\bar{\zeta}\gamma_5\psi'_{n\mu} - \bar{\psi}'_{n\mu}\gamma_5\zeta) + \frac{1}{2\sqrt{6}}(\bar{\zeta}\gamma_\mu\psi'_n - \bar{\psi}'_n\gamma_\mu\zeta), \\ \delta\varphi_n &= \sqrt{\frac{2}{3}}(\bar{\zeta}\gamma_5\psi'_n - \bar{\psi}'_n\gamma_5\zeta), \\ \delta\psi'_{n\mu} &= -\frac{i}{4}\gamma^{\rho\sigma}\omega_{n\mu\rho\sigma}\zeta - \frac{i}{4}\gamma^5\gamma^\sigma(\Delta h_{n\sigma\mu} - \partial_\sigma B_{n\mu}) \\ &\quad + F_{n\rho\sigma}(\gamma^{\rho\sigma}\gamma_\mu - 5\gamma^\rho\delta_\mu^\sigma + 5\gamma^\sigma\delta_\mu^\rho) + 2(\Delta A_{n\sigma} - \partial_\sigma\rho)(\gamma^5\gamma^\sigma\gamma_\mu - 5\gamma^5\delta_\mu^\sigma), \\ \delta\psi'_n &= \sqrt{\frac{3}{2}}\left\{-\frac{i}{8}\gamma^{\rho\sigma}\zeta(\partial_\rho B_{n\sigma} - \partial_\sigma B_{n\rho}) - \frac{i}{8}\gamma^5\gamma^\sigma\zeta(\Delta B_{n\sigma} - \partial_\sigma\varphi) \right. \\ &\quad \left. + F_{n\rho\sigma}\gamma^{\rho\sigma}\gamma_5 + 9(\Delta A_{n\sigma} - \partial_\sigma\rho)\gamma^\sigma\right\}, \\ \delta A_{n\mu} &= 80(\bar{\zeta}\psi'_{n\mu} + \bar{\psi}'_{n\mu}\zeta) - \frac{80}{\sqrt{6}}(\bar{\zeta}\gamma_\mu\gamma_5\psi'_n + \bar{\psi}'_n\gamma_5\gamma_\mu\zeta), \\ \delta\rho &= 80\sqrt{\frac{2}{3}}(\bar{\zeta}\psi'_n + \bar{\psi}'_n\zeta). \end{aligned} \quad (44)$$

Here  $\omega_{n\mu\nu\rho} = \frac{1}{2} \{\partial_\mu h_{n\rho\nu} - \partial_\rho h_{n\mu\nu}\}$  and the gauge condition (7) is used. This finishes the construction of linearized multisupergravity from discrete extra dimension. Note that such theory is free of ghosts (like the linearized multigravity) and most of symmetries of five-dimensional supergravity are respected. To address the consistency (ghosts presence) one needs to go beyond the linear level which is quite non-trivial task.

### 3 One-loop vacuum energy

Now the on-shell degrees of the freedom may be counted. As clear from (22),  $\Delta$  gives the mass. First one considers the massless particles. In the Lagrangian density (29), the massless particles are the graviton  $h_{n\mu\nu}$ , vector field  $B_{n\mu}$ , and scalar field  $\varphi_n$ . The on-shell degrees of the freedom are 2, 2, and 1, respectively. In the Lagrangian density (36) or (40), the massless particles are the complex Rarita-Schwinger field  $\psi'_{n\mu}$  and the Dirac fermion  $\psi'_n$ , whose on-shell degrees of the freedom are  $2 \times 2$  and  $2 \times 2$ . Finally in the Lagrangian density (41), the vector field  $A_{n\mu}$  and the scalar field  $\rho_n$  are massless and their physical degrees of the freedom are 2 and 1, respectively. Then in the massless sector, the total number of on-shell degrees of the freedom is 8 in the both of the bosonic and the fermionic sector. In the massive sector, several fields can be eliminated by the local symmetry transformation. In (29) by using the linearized general coordinate transformation (31), vector field  $B_{n\mu}$  and scalar field  $\varphi_n$  can be eliminated. The remaining field is massive graviton  $h_{n\mu\nu}$ , whose on-shell number of degrees of the freedom is 5 as  $h_{n\mu\nu}$  has spin 2. For the massive particles in (36), we may choose the gauge condition  $\psi'_n = 0$ , instead of (39), by using the local supersymmetry transformation (38). Then the remaining massive complex spin 3/2 particle  $\psi'_{n\mu}$  has the on-shell degrees of freedom  $4 \times 2$ . In (41), one may eliminate  $\rho_n$  in the massive sector by using the gauge transformation (42) and the remaining massive vector (spin 1) particle has 3 on-shell degrees of the freedom. Then even in the massive sector with common mass, the total number of on-shell degrees of the freedom is 8 in the both of the bosonic and the fermionic sector. As the on-shell degrees of the freedom in the bosonic sector coincide with those in the fermionic sector and now we are considering the flat background, the one-loop vacuum energy coming from the bosonic sector cancels with that from the fermionic sector.

For the fermionic sector, one may impose anti-periodic boundary condition for the discretized fifth dimension

$$\psi'_{n+N\mu} = -\psi'_{n\mu} , \quad \psi'_{n+N} = -\psi'_n . \quad (45)$$

Then instead of (21), the eigenvectors for  $\Delta$  are given by

$$\phi_{An}^M = \frac{1}{\sqrt{N}} e^{i \frac{2\pi n(M+\frac{1}{2})}{N}} , \quad M = 0, 1, \dots, N-1 \quad (46)$$

and their corresponding eigenvalues, by

$$\Delta \phi_{An}^M = i m_A^M \phi_{An}^M , \quad i m_A^M = \sum_{n=0}^{N-1} a_n e^{i \frac{2\pi n(M+\frac{1}{2})}{N}} . \quad (47)$$

As in (24),  $\phi_{An}^M$  satisfies the conditions of normalization and completeness:

$$\sum_{n=0}^{N-1} \phi_{An}^{M*} \phi_{An}^{M'} = \delta^{MM'} , \quad \sum_{M=0}^{N-1} \phi_{An}^{M*} \phi_{An'}^M = \delta_{nn'} . \quad (48)$$



If the fermionic particles obey the anti-periodic boundary condition, the (global) supersymmetry is explicitly broken, what becomes manifest in the mass spectrum. It is interesting that for continuous orbifold fifth dimension the antiperiodic boundary conditions may be interpreted as a discrete Wilson line breaking of supersymmetry (see, for instance [12]).

In general, the one-loop vacuum energy of the real scalar with mass  $m$  can be evaluated by the  $\zeta$ -function regularization[6]<sup>3</sup>:

$$V_{\text{eff}}^b = V_R^b(\mu) + \frac{m^4}{64\pi^2} \left( \ln \frac{m^2}{\mu^2} - \frac{3}{2} \right) . \quad (49)$$

Here  $\mu$  is introduced for the renormalization.  $V_R(\mu)$  is determined by the condition that  $V_{\text{eff}}$  should not depend on the arbitrary parameter  $\mu$ :

$$\mu \frac{dV_{\text{eff}}(\mu)}{d\mu} = 0 . \quad (50)$$

As we are now considering the flat background, the contribution to the one-loop vacuum energy from each of the bosonic degrees of freedom is given by (49). On the other hand, the contribution  $V_{\text{eff}}^f$  from each of the fermionic degrees of freedom is different from that from bosonic ones by sign:

$$V_{\text{eff}}^f = -V_{\text{eff}}^b . \quad (51)$$

If the fermionic particles satisfy the periodic boundary condition, the mass spectrum and the degrees of freedom of the fermionic particle are identical with those of the bosonic particles. Then the one-loop vacuum energy vanishes, which is also a signal of the supersymmetry. On the other hand, if the boundary condition for fermionic fields is anti-periodic, the vacuum energy does not vanish in general. As an example, we consider  $N = 3$  case in (27). Then for the bosonic sector, the mass  $m_b$  can be

$$m_b^2 = (0, m_1^2, m_1^2) . \quad (52)$$

For the fermionic sector, by using (47), we find the mass  $m_f$  for the fermionic sector is given by

$$|m_f|^2 = \left( \frac{m_1^2}{3}, \frac{4m_1^2}{3}, \frac{4m_1^2}{3} \right) . \quad (53)$$

Due to the anti-periodic boundary condition, there is no massless fermionic particle. The total effective potential  $V_{\text{eff}}$  is given by

$$\begin{aligned} V_{\text{eff}} &= V_R(\mu) + \frac{16m_1^4}{64\pi^2} \left( \ln \frac{m_1^2}{\mu^2} - \frac{3}{2} \right) \\ &\quad - 8 \left\{ \frac{2m_1^4}{9 \cdot 64\pi^2} \left( \ln \frac{m_1^2}{3\mu^2} - \frac{3}{2} \right) + \frac{16m_1^4}{9 \cdot 64\pi^2} \left( \ln \frac{4m_1^2}{4\mu^2} - \frac{3}{2} \right) \right\} \\ &= V_R(\mu) - \frac{m_1^4}{36\pi^2} (16 \ln 2 - \ln 3) . \end{aligned} \quad (54)$$

Eq.(50) gives  $V_R(\mu) = 0$  and

$$V_{\text{eff}} = -\frac{m_1^4}{36\pi^2} (16 \ln 2 - \ln 3) < 0 . \quad (55)$$

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<sup>3</sup>The related discussion of matter Casimir effect in deconstruction maybe found in [16]

In the lattice field theory,  $a_1$  in (27) is related with the lattice spacing  $a$  by  $a_1 = \frac{1}{2a}$ . As a result,  $m_1 = \frac{\sqrt{3}}{2a}$ . If we regard the sites on the lattice with the branes in the extra dimension, the lattice spacing  $a$  may correspond to the distance between the branes.

In getting (55), we have assumed that the bosonic particles obey the periodic boundary condition as we consider the lattice on the circle  $S^1$ . However, taking the lattice on the orbifold obtained by dividing  $S^1$  with discrete subgroup  $Z_2$  (with the coordinate on the circle as  $\theta$ ,  $0 \leq \theta < 2\pi$ ,  $S^1/Z_2$  can be obtained by identifying  $\theta$  with  $\theta + \pi$ ), the bosonic particle can obey the anti-periodic boundary condition. Thus, for the anti-periodic boundary condition for the bosonic particles and the periodic one for the fermionic particles, the sign of the vacuum energy (55) is reversed and one obtains the effective positive cosmological constant (dark energy) [6], which may explain the current universe acceleration.

## 4 Discussion

The resulting theory contains multigravitons and gravitinos. In this sense, we may call it multisupergravity. However, this is linearized model which does not include the interaction. In order to obtain the complete supergravity theory with the interaction, we may start from the local supersymmetry transformation which is given by combining the linearized local supersymmetry transformation (38) with the transformation (45) after replacing the constant spinor parameter  $\zeta$  with the local parameter  $\eta_n$ . The variation of the total Lagrangian density  $\mathcal{L} = \mathcal{L}_E + \mathcal{L}_{RS} + \mathcal{L}_U$  is not invariant but proportional to  $\partial_\mu \eta_n$  or  $\partial_\mu \bar{\eta}_n$  up to the total derivative. One may add the term obtained by replacing  $\partial_\mu \eta_n$  ( $\partial_\mu \bar{\eta}_n$ ) in the variation with  $-\psi'_{n\mu} - (\bar{\psi}'_{n\mu})$  to the action as a counterterm. The counterterm includes interaction in general. The modified Lagrangian is not still invariant in general but its variation is proportional to  $\partial_\mu \eta_n$  or  $\partial_\mu \bar{\eta}_n$  up to the total derivative again. More counterterms may be added. This is nothing but the Noether method. If the procedure ends up by the finite number of the iterations, the complete supergravity model results. In general, however, the procedure does not end up with the finite iteration. Instead, as in [4], we may start from the  $N$ -copies of some kind of four-dimensional supergravity (with interaction) and introduce matter multiplet linking the copies. As a result, one may construct an interacting theory with massive graviton and gravitino(s). Linearizing such a theory, one gets the model of the sort discussed in this paper. It could be that complete discretized supergravity (for which the theory under consideration is just the first, preliminary step) may be useful to solve the consistency problem of multigravity (see, however, example of ghost free bi-gravity in [13]). Unfortunately, its construction is highly non-trivial problem. Note, however, that after first submission of this work to hep-th the very interesting approach which admits the construction of complete multisupergravity in terms of superfields has been developed [15].

Although the five-dimensional linearized discretized supergravity is considered, the generalization to higher dimensions may be done. For example, if we start from the eleven-dimensional supergravity [14] and replace the eleventh direction with the discrete lattice, we may obtain the ten-dimensional supergravity theories with multigravitons and multigravitinos. This may open new interesting connection between M-theory and ten-dimensional supergravities (strings).

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## A $\gamma$ -matrices

Here, the conventions and the basic formulae for the five-dimensional  $\gamma$ -matrices are summarized.

The definition of the  $\gamma$ -matrices is

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} . \quad (56)$$

$\gamma^0$  is anti-hermitian,  $\gamma^i$  ( $i = 1, 2, 3, 4$ )'s are hermitian and  $\gamma_0 (\gamma^\mu)^\dagger \gamma_0 = \gamma^\mu$ . For the spinor field  $\psi$ , we define  $\bar{\psi} \equiv i\psi^\dagger \gamma_0$ . As a result,  $(\bar{\psi}_1 \psi_2)^\dagger = \bar{\psi}_2 \psi_1$ ,  $(\bar{\psi}_1 \gamma^\mu \psi_2)^\dagger = -\bar{\psi}_2 \gamma^\mu \psi_1$ , etc. We also define  $\gamma^{\mu\nu}$  and  $\gamma^{\mu\nu\rho}$  by

$$\begin{aligned} \gamma^{\mu\nu} &\equiv \frac{1}{2} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) , \\ \gamma^{\mu\nu\rho} &\equiv \frac{1}{6} (\gamma^\mu \gamma^\nu \gamma^\rho + \gamma^\nu \gamma^\rho \gamma^\mu + \gamma^\rho \gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu \gamma^\rho - \gamma^\mu \gamma^\rho \gamma^\nu - \gamma^\rho \gamma^\nu \gamma^\mu) . \end{aligned} \quad (57)$$

where

$$\gamma^{\mu\nu\rho} = -i\epsilon^{\mu\nu\rho\sigma\tau} \gamma_{\sigma\tau} . \quad (58)$$

Here  $\epsilon^{\mu\nu\rho\sigma\tau}$  is rank 5 anti-symmetric tensor and  $\epsilon^{01234} = 1$ . The following relations are necessary to show the invariance under the global supersymmetry transformation (11):

$$\begin{aligned} \gamma^{\eta\zeta} \gamma^\sigma &= \gamma^{\eta\zeta\sigma} + \gamma^\eta \gamma^{\sigma\zeta} - \gamma^\zeta \gamma^{\sigma\eta} , \\ \gamma^{\mu\nu\rho} \gamma^{\sigma\tau} &= -4i\epsilon^{\mu\nu\rho\sigma\tau} - 4(-\eta^{\rho\sigma} \eta^{\mu\tau} \gamma^\nu + \eta^{\mu\sigma} \eta^{\rho\tau} \gamma^\nu - \eta^{\mu\sigma} \eta^{\nu\tau} \gamma^\rho + \eta^{\nu\sigma} \eta^{\mu\tau} \gamma^\rho \\ &\quad - \eta^{\nu\sigma} \eta^{\rho\tau} \gamma^\mu + \eta^{\rho\sigma} \eta^{\nu\tau} \gamma^\mu) \\ &\quad - 2(\eta^{\mu\sigma} \gamma^{\nu\rho\tau} + \eta^{\nu\sigma} \gamma^{\rho\mu\tau} + \eta^{\rho\sigma} \gamma^{\mu\nu\tau}) + 2(\eta^{\mu\tau} \gamma^{\nu\rho\sigma} + \eta^{\nu\tau} \gamma^{\rho\mu\sigma} + \eta^{\rho\tau} \gamma^{\mu\nu\sigma}) \\ &\quad + 2i\epsilon^{\mu\nu\rho\eta\sigma} \gamma_\eta^\tau - 2i\epsilon^{\mu\nu\rho\eta\tau} \gamma_\eta^\sigma , \\ \gamma^{\mu\nu\rho} \gamma^\sigma &= -4(\eta^{\mu\sigma} \gamma^{\nu\rho} + \eta^{\rho\sigma} \gamma^{\mu\nu} + \eta^{\sigma\nu} \gamma^{\rho\mu}) + 2i\epsilon^{\mu\nu\rho\sigma\tau} \gamma_\tau . \end{aligned} \quad (59)$$

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